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Gillespie, P. D., Gaida, D., Hung, P. C., Kee, R. J., & McLoone, S. F. (2016). A Bias Compensated Cross-Relation approach to Thermocouple Characterisation. *IFAC-PapersOnLine*, 49(5), 43-48.
<https://doi.org/10.1016/j.ifacol.2016.07.087>

Published in:
IFAC-PapersOnLine

Document Version:
Peer reviewed version

Queen's University Belfast - Research Portal:
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A Bias Compensated Cross-Relation approach to Thermocouple Characterisation

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Abstract: The measurement of fast changing temperature fluctuations is a challenging problem due to the inherent limited bandwidth of temperature sensors. This results in a measured signal that is a lagged and attenuated version of the input. Compensation can be performed provided an accurate, parametrised sensor model is available. However, to account for the influence of the measurement environment and changing conditions such as gas velocity, the model must be estimated *in-situ*. The cross-relation method of blind deconvolution is one approach for *in-situ* characterisation of sensors. However, a drawback with the method is that it becomes positively biased and unstable at high noise levels. In this paper the cross-relation method is cast in the discrete-time domain and a bias compensation approach is developed. It is shown that the proposed compensation scheme is robust and yields unbiased estimates with lower estimation variance than the uncompensated version. All results are verified using Monte-Carlo simulations.

Keywords: Cross-relation, Blind sensor characterisation, temperature measurement.

1. INTRODUCTION

Accurate measurement of temperature is necessary and in some cases, critical, in many industrial and scientific applications. In the automotive industry, for example, an accurate measurement of exhaust gas temperature is required for onboard diagnosis of catalyst malfunction (Kee et al., 2006), and to provide insight into engine combustion which can be used to evaluate performance. In this and other environments where the temperature is changing rapidly, fast temperature measurement can be performed using advanced techniques such as coherent anti-stokes spectroscopy, laser induced fluorescence, and infrared pyrometry. However, the instrumentation required for these is expensive, difficult to calibrate and maintain and is therefore not practical for general applications. A practical alternative is to use a thermocouple. These provide an inexpensive and robust method of measuring temperature over a wide range and at low cost. However, like all sensors, thermocouples have a limited bandwidth and are effectively low-pass filters. Their bandwidth is determined primarily by the wire diameter, and to a lesser degree by the velocity of the surrounding gas (Kee et al.,

2006). A consequence of this is that when the frequency of temperature fluctuations exceeds the bandwidth of the thermocouple, the measured output is an attenuated and delayed version of the input.

The most obvious approach to reduce the measurement error is to make the wire diameter smaller, which in theory should increase the effective sensor bandwidth. Practically speaking, however, this is not a viable solution since it would result in a sensor that is mechanically fragile and unable to withstand harsh measurement environments. An alternative approach is to employ software-based compensation techniques. A requirement of this approach is that a dynamic model of the sensor is available and the sensor model parameters are known prior to compensation (Hung et al., 2005). Furthermore, the model parameters should take the measurement environment into account and any other factors that may influence the dynamic characteristics of the sensor.

A loop current step response (LCSR) test is a generally accepted method to estimate the parameters of a sensor model *in-situ* (Hashemian and Petersen, 1992). However, it has some major disadvantages that render it unsuitable

in certain applications. The most significant problem is that the test is intrusive and the relatively high levels of heating can potentially disrupt the process being measured (Tan et al., 2006). Furthermore, it is time consuming since the test must be regularly performed to adapt to changing conditions and can therefore represent significant costs. Consequently, it is desirable to have a non-intrusive method to estimate the model parameters before compensating the sensor output. Since a measurable input signal is not available using a non-intrusive method, the model identification problem becomes one of identifying the model using only measured output data. This is known as a blind identification problem, or in this application, blind sensor characterisation.

In 1936, Pfriem, a German engineer proposed a solution to this problem (Pfriem, 1936). His solution specified the use of two sensors of known model structure, each with different dynamic characteristics and placed in the measurement environment such that they both experience the same thermal field. He then postulated that it would be possible to estimate both sensor models using only output measurements from the sensors. Since then, numerous approaches for two-thermocouple characterisation have been proposed based on the principle and assumptions of the approach proposed by Pfriem. A range of time-domain methods have been presented by Tagawa et al. (1998), Kee et al. (1999), O'Reilly et al. (2001) and Kar et al. (2004). Forney and Fralick (1994) and later Tagawa et al. (2003) then tackled the problem in the frequency-domain, thereby avoiding the numerical problems associated with estimating derivatives that were experienced in the time-domain. In recent years, with the growth and development in digital based instrumentation, a number of methods were developed in the discrete-time domain that take account of sampled data. The authors of Hung et al. (2005) were the first to cast the sensor characterisation problem in the system identification field and demonstrated an approach using difference equations that provides superior performance to other approaches in the literature. In an attempt to improve on the performance of the difference equation approach, Hung et al. (2007) proposed a new approach for sensor characterisation using a technique known as the cross-relation approach, initially proposed by Liu et al. (1993) for application to communication channel equalisation. The approach is based in the continuous-time domain and involves estimating the time constants of the sensor models directly by minimising an error cost function. For simulation purposes the sensor models are represented as first-order continuous-time transfer functions and the output is computed by numerical integration. The main limitation of this approach is its limited resilience to high noise levels. The noise contribution has the effect of distorting the shape of the cost function around the local optimum in such a way that it becomes ill-posed as the noise level increases and characterisation is no longer possible. In addition, the distortion imposed on the cost function results in biased estimates.

In this paper we present a methodology for advancing the work of Hung et al. (2007) by casting the problem explicitly in the discrete-time domain. In particular, we develop a novel bias compensation technique that yields

unbiased parameter estimates and significantly improves the robustness of the method to measurement noise.

2. BLIND CHARACTERISATION

The cross-relation method proposed by Liu et al. (1993) is based on the principle of commutativity and the assumption that the systems to be identified are linear. A discrete-time formulation of the sensor characterisation problem using the cross-relation approach is illustrated in Fig. 1. Both sensors, which are represented by the transfer functions, G_1 and G_2 , are assumed to experience the same input temperature, T_g^k and the outputs, T_{m1}^k and T_{m2}^k are linearly related by their responses. The sensor outputs are then collected and passed through the synthetic sensor models, \hat{G}_2 and \hat{G}_1 which are represented by discrete-time first-order transfer functions of the form

$$\hat{G}_1(z^{-1}) = \frac{\hat{b}z^{-1}}{1 - \hat{a}z^{-1}} \quad \text{and} \quad \hat{G}_2(z^{-1}) = \frac{\hat{d}z^{-1}}{1 - \hat{c}z^{-1}} \quad (1)$$

to produce the outputs T_{m12}^k and T_{m21}^k . Here, \hat{G}_1 and \hat{G}_2 are estimates of the true sensor transfer functions, G_1 and G_2 . The unknown model parameters to be estimated are

$$\phi = [a, b, c, d]. \quad (2)$$

By virtue of the commutative property of linear systems which states that, for noise free sensor outputs,

$$T_{m1} = G_1 \otimes T_g \quad \text{and} \quad T_{m2} = G_2 \otimes T_g \quad (3)$$

where \otimes indicates convolution, it follows that

$$G_1 \otimes \hat{G}_2 = G_2 \otimes \hat{G}_1. \quad (4)$$

Hence, the cross-relation method estimates the sensor models by adjusting ϕ such that the difference between the outputs of the two signals paths in Fig. 1 is minimised.

It is assumed that both sensors have been properly calibrated to have unity gain so that $b = 1 - a$ and $d = 1 - c$. Under these conditions, a 2-D mean-squared-error (MSE) cost function, defined as

$$J_{CR}(\hat{a}, \hat{c}) = \frac{1}{N} \sum_{k_0+1}^{k_0+N} (e^k)^2, \quad \forall \hat{a}_1, \hat{c}_1, \quad (5)$$

where

$$e^k = T_{m12}^k - T_{m21}^k \quad (6)$$

can be minimised to give estimates of a and c . Since the initial conditions for the sensors are not known in practice, the first k_0 data samples are omitted to avoid output mismatch due to initial transients. Here, k_0 is chosen to be greater than 5 times the largest sensor time constant.

While it is apparent that the cross-relation cost function $J_{CR}(\hat{a}, \hat{c})$ will be zero when $\hat{a} = a$ and $\hat{c} = c$, in practice it will not be possible to obtain an exact match between

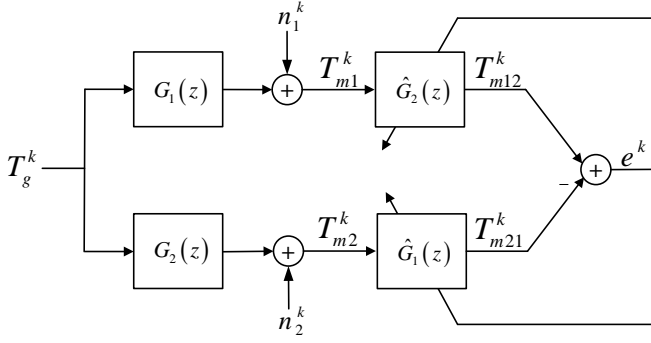


Fig. 1. Two-sensor cross-relation characterisation

T_{m12}^k and T_{m21}^k due to the presence of measurement noise on the sensor signals, modelling inaccuracies and violations of the assumption that both sensors are experiencing identical thermal conditions. Consequently, the local minimum will not necessarily be exactly zero, even for the correct parameter choice. In other words $J_{CR}(a, c) \neq 0$. However, despite such discrepancies, minimising the cost function will yield estimates of the unknown parameters.

Alternative approaches, such as the difference equation approach presented in Gillespie et al. (2015) also convert sensor characterisation into an optimisation problem. For first-order models, the difference equation approach produces a quadratic cost function that can be solved using linear least-squares optimisation techniques. However, the cross-relation cost function is non-quadratic and cannot be solved using linear optimisation techniques. A 3-D surface plot and contour map of the cost function J_{CR} given in in Fig. 2 shows that it is multimodal in nature with a second minimum when $\hat{a} = \hat{c} = 1$. This corresponds to the situation where the sensor time constants $\hat{\tau}_1 = \hat{\tau}_2 \rightarrow \infty$. In these circumstances the sensors, which are modelled as low-pass filters, are effectively open circuited and hence their difference will always be zero. It should be noted that the minimum located at $\hat{a} = \hat{c} = 1$ exists independent of noise conditions, any modelling violations or assumptions about the thermal fields experienced by both sensors, and is in fact, the global minimum, i.e. $J_{CR}(1, 1) = 0$.

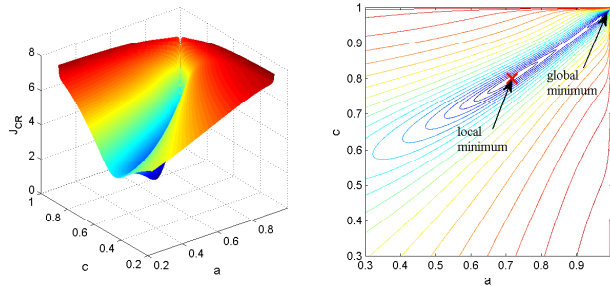


Fig. 2. (a) Cost function for noiseless sensor measurements; (b) contour map of cost function showing local and global minimum

An alternative way to view the details of the cost function J_{CR} is to consider the trajectory of the minimum of \hat{a} as a function of \hat{c} and vice versa, that is

$$J(\hat{c}) = \min_{\hat{a}} J_{CR}(\hat{a}, \hat{c}) \quad \forall \hat{a} \in [0, 1] \quad (7)$$

$$J(\hat{a}) = \min_{\hat{c}} J_{CR}(\hat{a}, \hat{c}) \quad \forall \hat{c} \in [0, 1]. \quad (8)$$

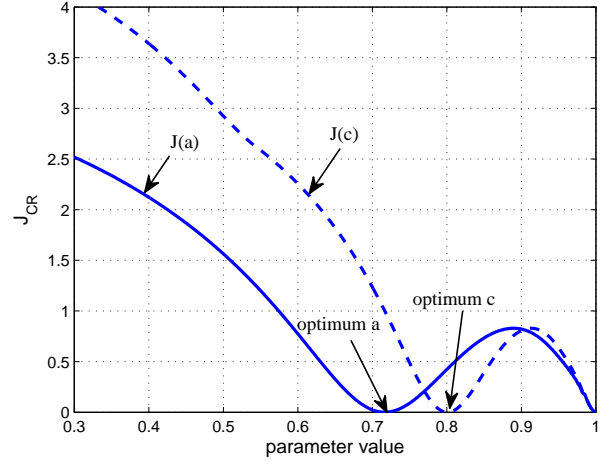


Fig. 3. Trajectories of minimum value of J_{CR} with respect to \hat{a}_1 and \hat{c}_1 respectively

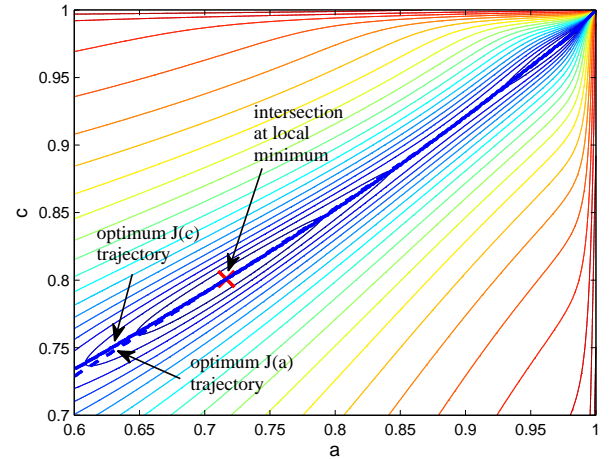


Fig. 4. A portion of Fig. 2(a) in the vicinity of the local minimum showing the two trajectory curves superimposed and intersecting at the local minimum

These are plotted in Fig. 3 for the cost function given in Fig. 2. This cost function was produced using data samples collected using the simulation setup introduced in Section 3. Each 1-D plot has a clearly defined local minimum for the correct choice of \hat{a} and \hat{c} and a global minimum when both parameters are equal to one. The relationships between the trajectories and minima can be visualised more clearly if they are superimposed on a contour map, as illustrated in Fig. 4. Note that trajectories intersect at the local minimum and global minimum, but are not necessarily coincident elsewhere.

3. SIMULATION SETUP

To confirm the accuracy of the cross-relation scheme for sensor characterisation, simulations were performed based on the block diagram shown in Fig. 5. Both sensors were

modelled as digital low-pass filters where G_1 has a larger bandwidth. Zero-mean, white Gaussian noise sequences were added to the outputs of the filters to represent measurement noise. In all simulations, noise of equal power was added to each sensor, i.e. $\text{var}(n_1) = \text{var}(n_2)$.

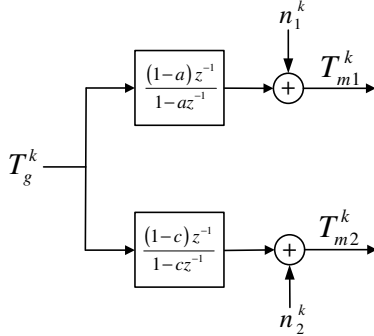


Fig. 5. Block diagram of two thermocouple sensors

A fluctuating gas or fluid temperature signal was represented as a multitone sinusoidal signal as shown in Fig. 6. Each data set consists of 2000 samples which were collected at a sample frequency of 5 Hz before the first 30 samples, which contain initial condition transients, were removed. The parameters of the discrete-time models are $a = 0.72$ and $c = 0.8$ for sensor 1 and 2, respectively.

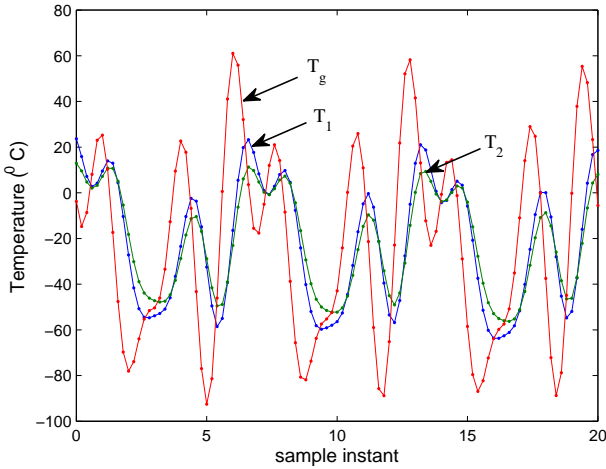


Fig. 6. Simulated multitone temperature fluctuations

4. EFFECT OF NOISE

Fig. 7 shows a set of simulated 1-D cost functions produced using sensor measurements with different levels of additive noise, each defined according to a specific noise power σ^2 . The noise power increases linearly from 0 to 12 and in each case the gradient based optimisation algorithm employed is initialised close to 0. It is clearly evident from the plots that as the noise power increases, the local minimum gradually becomes shallower until it eventually disappears. If no local minimum point exists within the search range, then an unbounded optimisation algorithm will tend to find the global minimum. However, with constraints added the algorithm will stop at the upper bound of the search interval. Generally speaking,

as the noise level increases the local minimum becomes increasingly more shallow and positively biased. This is a direct consequence of the contribution of the noise component on the input signal, that is, the noise present on the sensor signals.

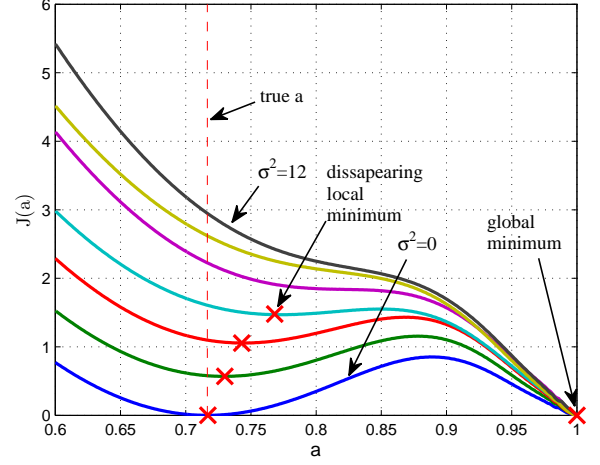


Fig. 7. Cost function $J(\hat{a})$ for noise powers $\sigma^2=0$ to 12

5. BIAS COMPENSATION

To address this issue we propose the introduction of a bias compensation technique, to correct for the bias introduced by the noise by removing an estimate of the noise contribution from the cost function before optimisation. By assuming that the system is linear and superposition applies, an analysis by McLoone et al. (2008) shows that the variance of the cross-correlation error in (6) can be expressed as

$$E\{(\tilde{e}^k)^2\} = E\{(e^k)^2\} + E\{(\tilde{n}_1^k)^2\} + E\{(\tilde{n}_2^k)^2\}. \quad (9)$$

The first term equates to the noise free mean-squared-error due to an incorrect choice of parameters. The second and third terms are due to filtered measurement noise. In other words, they are the result of passing the noise contributions of the sensor signals n_1 and n_2 through the synthetic models. It can be shown (see Appendix A) that when the noise is assumed to be zero-mean, white noise with power σ^2 , the variance of the noise at the output of the synthetic model is given by

$$E\{(\tilde{n}_1^k)^2\} = \frac{1-c}{1+c}\sigma^2, \quad E\{(\tilde{n}_2^k)^2\} = \frac{1-a}{1+a}\sigma^2. \quad (10)$$

Hence the noise contribution to the cross-correlation cost function can be expressed as

$$N_{CR} = \left(\frac{1-a}{1+a} + \frac{1-c}{1+c} \right) \sigma^2. \quad (11)$$

The cost function can then be expressed as

$$J_{CR}(\hat{a}, \hat{c}) = J_{CR}^{NF} + N_{CR} \quad (12)$$

where J_{CR}^{NF} is the noise free component. A bias compensated version of the cost function $J_{CR}^{\sigma^2}$ can be then be defined as

$$J_{CR}^{\sigma^2} = J_{CR} - \left[\left(\frac{1-a}{1+a} + \frac{1-c}{1+c} \right) \sigma^2 \right]. \quad (13)$$

The results obtained after applying the compensation given by (13) to the cost functions in Fig. 7 is shown in Fig. 8.

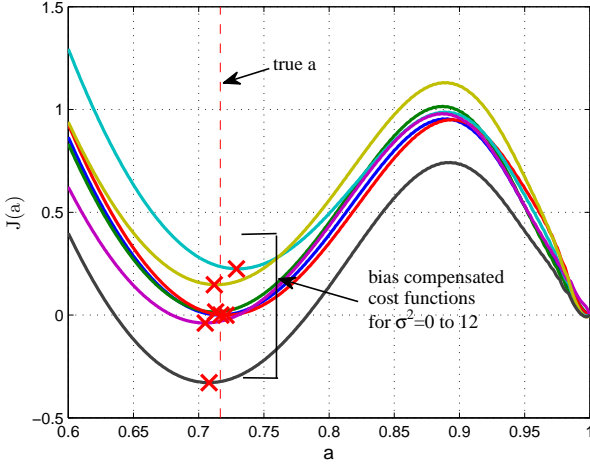


Fig. 8. Compensated cost functions $J(\hat{a})$ for noise power $\sigma^2=0$ to $\sigma^2=12$

Clearly, the compensation scheme has significantly improved the shape of the cost function in the vicinity of the local minimum. For each noise power considered, the local minimum of the compensated cost function is now distinctly defined. An optimisation algorithm would have little difficulty in identifying the local minimum of the bias compensated cost functions, provided it is correctly initialised. In some cases the local optimum becomes negative, although practically this will not affect the solution. The mean and standard deviation of \hat{a} for the bias compensated and uncompensated cost function is shown in Fig. 9. The statistical metrics are computed from a Monte-Carlo simulation of 50 runs for a range of noise powers. While the optimum value of a in the uncompensated cost function becomes increasingly positively biased as the noise power increases, for the compensated cost function, it remains unbiased for the noise powers considered here.

In practice, the noise power is unknown and must be estimated. Fig. 10 demonstrates how the estimation accuracy of parameter a depends on the estimated noise power, used for compensation. Only parameter a is shown, since a similar pattern is observed for parameter c . The plots show the estimation error post compensation for two different sensor signal noise powers ($\sigma^2 = 10$ and $\sigma^2 = 5$, respectively) as a function of estimated noise power used for compensation. Note that as expected, when the noise power used for compensation matches the actual noise power, $\hat{a} = a$.

As the estimated noise power increases beyond the correct value, \hat{a} becomes increasingly negatively biased. When the

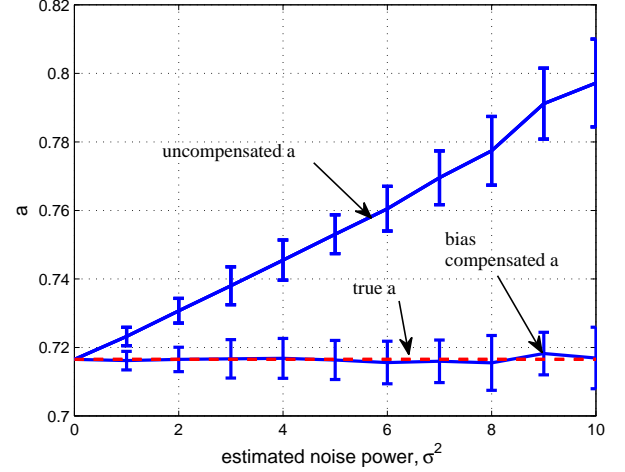


Fig. 9. The mean and standard deviation of \hat{a} for the bias compensated and uncompensated cross-relation estimates as a function of noise power

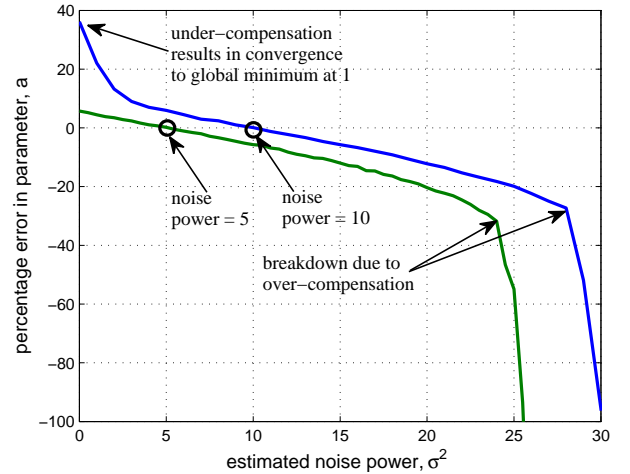


Fig. 10. Estimation error as a function of noise power

estimated noise power is too high, the scheme breaks down completely yielding $\hat{a} = 0$ as the parameter estimate. This arises because over compensation 'tilts' the cost function to the point where $(0, 0)$ is the lowest value in the range $[0, 1]$.

The effect of under-compensation is that \hat{a} becomes increasingly positively biased as the estimated noise power decreases below the correct value. Note that for the scenario where $\sigma^2 = 10$, the compensation breaks down when the estimated noise power is too small. On the other hand, for the scenario where $\sigma^2 = 5$, the bias increases linearly as $\sigma^2 \rightarrow 0$, with no apparent break down. This behaviour is due to the fact that the local minimum exists for the uncompensated cost function for the scenario where $\sigma^2 = 5$. Hence, applying any amount of compensation has the effect of reducing the bias of \hat{a} but not for the scenario where $\sigma^2 = 10$ (see Fig. 7).

6. CONCLUSION

A novel bias compensation scheme for cross-relation based sensor characterisation has been presented. The significance of this approach is that accurate sensor characterisation is still possible, even when the sensor measurements are heavily contaminated with noise, provided an accurate estimate of the noise power is available. The method is also relatively robust to errors in the noise power estimate used for compensation. Results for the simulated case study show that a 50% error in σ^2 results in an error in a of approximately 6% when $\sigma^2 = 10$ and 3% when $\sigma^2 = 5$.

Appendix A. DERIVATION OF BIAS COMPENSATION TERM

Consider the synthetic sensor model represented by the difference equation

$$y^k = ay^{k-1} + (1-a)u^{k-1} \quad (\text{A.1})$$

where y^k is the output and u^k is the input. If the input is a zero-mean, Gaussian white noise sequence, r^k , with noise power σ^2 , that is

$$r^k = N(0, \sigma^2), \quad E[(r^k)] = 0, \quad E[(r^k)^2] = \sigma^2, \quad (\text{A.2})$$

then the sensor output at the k^{th} sample instant can be represented as

$$y^k = a^k y_0 + (1-a) \left(\sum_{i=1}^k a^{k-i} r^{i-1} \right) \quad (\text{A.3})$$

where y_0 is the initial output. Assuming zero initial conditions, the variance of the output is given by

$$\begin{aligned} E[(y^k)^2] &= (1-a)^2 \sigma^2 \sum_{j=0}^{k-1} (a^2)^j \\ &= (1-a)^2 \sigma^2 \frac{1 - (a^2)^k}{1 - a^2}. \end{aligned} \quad (\text{A.4})$$

This reduces to

$$E[(y^k)^2, a] = \frac{1-a}{1+a} \sigma^2 (1 - a^{2k}). \quad (\text{A.5})$$

As $k \rightarrow \infty$, $a^{2k} \rightarrow 0$ provided $|a| < 1$. Hence,

$$E(y_\infty^2, a) = \frac{1-a}{1+a} \sigma^2. \quad (\text{A.6})$$

ACKNOWLEDGEMENTS

The first author gratefully acknowledges the financial support of a DEL PhD studentship from the Department for Employment and Learning, Northern Ireland.

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